Comments on Nonlocality in Deformed Special Relativity, in reply to arXiv:1004.0664 by Lee Smolin and arXiv:1004.0575 by Jacob et al.

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Abstract

It was previously shown that models with deformations of special relativity that have an energy-dependent yet observer-independent speed of light suffer from non-local effects that are in conflict with observation to very high precision. In a recent paper it has been proposed that these paradoxa arise only in the classical limit and can be prevented by an ad-hoc introduction of a quantum uncertainty that would serve to hide the nonlocality. We will show here that the proposed ansatz for this resolution is inconsistent with observer-independence and, when corrected, is in agreement with the earlier argument that revealed the troublesome nonlocality. We further offer an alternative derivation for the energy-dependent speed of light in the model used.

1 What the argument is about

It has been claimed that deformations of special relativity (DSR) [1] make it possible to introduce an energy-dependent speed of light in position space while preserving observer-independence. Or, to be exact, since there is no (agreed upon) formulation of DSR in position space the energy-dependent speed of light $\tilde{c}(E)$ that one finds in these models in momentum space has been used as a speed in position space [2]. In particular it has been used to make predictions for a possible time-delay in the arrival time of high energetic photons from distant gamma ray bursts that might be on the edge of detection. As an observable prediction of quantum gravitational effects, this has received a lot of attention [3, 4, 5, 6].

It goes without saying that the lacking derivation of the photon's propagation in position space is a severe shortcoming of the model and raises strong doubts about the use of the prediction to begin with. Moreover, in [7] it has been argued that even absent a derivation of the propagation in position space, it can be concluded that an energy-dependent and observer-independent speed of light of the sort that leads to the predictions made is in conflict with observations already, at least to first order in energy over Planck mass, E/m_p .

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It should be emphasized that in [7] it has not been shown that DSR does have an energy-dependent speed of light in position space¹. The logic of the argument presented in [7] is instead to assume that there is indeed such an energy-dependent speed of light that would lead to an observable time-delay in high energetic gamma ray bursts, and that this effect would also be observer-independent. Then it can be shown that this leads to strongly non-local effects that would long have been observed. This means in return, since we have not observed such effects, DSR either does not have an energy-dependent speed of light in position space, or it does in which case it is not compatible with reality. Either way one can conclude that DSR cannot give rise to observable time-delays in gamma ray bursts in contrast to the predictions made.

Of course this conclusion depends on the assumptions made, the details can be found in [7]. It should also be mentioned that some problems with locality in DSR had been pointed out previously by Unruh and Schützhold [9] but the nonlocality had not been quantified. It had been claimed by Amelino-Camelia that such nonlocal effects "can be safely neglected" [10].

The reason for the arising nonlocality is, in short, as follows. In special relativity there is exactly one speed that is observer-independent, that is the usual, constant, speed of light c. In DSR now photons with different energies travel with different speeds, depending on their energy. If this speed of light $\tilde{c}(E)$ is observer-independent, an observer who measures the energy E' has to find $\tilde{c}(E')$ for its speed. It is easy to see that this is not possible with usual Lorentz-transformations. To achieve the invariance of the speed of light one has to use modified Lorentz-transformations. These transformations then have to depend on the energy of the particle.

Leaving aside for a moment the interpretational problems with a change of reference frame depending on an energy (which?), this causes another problem: Since the crossing of worldlines defines events in space-time, using these modified, energy-dependent, transformations changes the location in which two world-lines meet relative to the special relativity case. For two worldlines this might not seem too worrisome since there is no absolute meaning to position anyway and (in 1+1 dimensions) the two lines will generically meet in some point. But if one considers three worldlines that in one frame meet in one point, then the use of the modified transformation will generically cause this point to split up in three different points. Or rather, there is then no such thing as "one point" – it's an ill-defined concept. This is a consequence of the three worldlines transforming differently now. In the special relativity case, three lines that meet in one point in one reference frame will meet in one point in all reference frames.

How much the point splits up depends on the distance the particles have traveled. This is because the modified transformations change, diagrammatically speaking, the angle of the worldline relative to the usual Lorentz-transformation, thus the total change adds up over the distance. This is the same reason why the effect has been claimed to be observable in the first place. It can be shown that the splitting of the point for realistic distances and particle energies can be as large as $\approx 1 \, \mathrm{km}$ just by changing from the Earth restframe to

¹It has been claimed in [8] that the speed of massless particles is constant in DSR.

a satellite in Earth orbit. One might have expected some sort of deviation from the usual space-time picture, but it should not happen on such macroscopic scales.

This is the simple case of the locality problem that has in [7] been referred to as Version 2.0. In Version 2.1 it has further been considered the option that DSR in position space also causes a spread of a (usually dispersionless) photon wave-packet. This spread has then to be compared to the splitting of the points to see whether it can hide the nonlocality. It has been shown that while such a spread vastly improves the problem, it is still not possible to hide the nonlocality in all reference frames, and that even moderate boosts (up to $\gamma \approx 20$) are sufficient to reveal the nonlocality again.

2 The proposed solution

In a recent paper by Lee Smolin [11] it has now been proposed an ansatz for quantum uncertainties that could potentially address the problem of nonlocality in DSR. The idea is that the nonlocality, the splitting-up of the point, is entirely hidden by quantum position uncertainty. What one needs is that the uncertainty in a particle's position is so large that the probability of it being at any of the three split-up points is about the same in all restframes. Then, for what the particle is concerned, the three points could not be resolved and it would be appropriate to understand them as one large point, removing the inconsistency.

In [11] it has not been examined the three-particle case discussed in [7] and it was left open for future studies whether the proposed ansatz would indeed solve the problem. Before we look into the proposal of [11], let us note that even if the attempt to hide the nonlocality by additional uncertainty was successful, it would still mean that previously made predictions for gamma ray bursts were wrong (as also pointed out in [11]). The effect would then not present itself in an either earlier or later arrival of highly energetic photons, but instead in a stochastic spread over arrival times where the width of the spread depends on the energy.

The ansatz made in [11], see Eqs.(44,45), is

where we have set $\hbar=1$ and $l_{\rm p}=1/m_{\rm p}$ is the Planck length. Note that this ansatz is being made in one particular frame, in this case Alice's. Here, E and p are the energy and momentum as measured by Alice. |a| is the absolute value of an earlier introduced position vector a^i . Since this vector can be arbitrarily chosen, it is unclear what its meaning is. However, the ansatz later used for the spread of the wavefunction is that |a| is actually the distance the particle travelled, see the later remark in [11], after Eq. (65).

That the ansatz Eq.(1) should solve the problem pointed out in [7] is puzzling, since it is the same spread of the wave-packet that has been examined there (see Eq.(13) [7] and the paragraph thereafter). To be more precise it is the same spread when one makes the approximation that the initial width of the photon's wave-function in momentum space is approximately its energy $\Delta p \approx |p|$. It has been pointed out in [7], that this corresponds to a

quite badly localized particle already. This situation was examined in [7] because it is the most optimistic case when one tries to make sense of DSR, and the most challenging one if one tries to show it inconsistent. It was probably studied in [11] for the same reasons. Either way, it was shown in [7] that even when one considers such an already badly localized wave-packet the spread cannot entirely hide the nonlocality. So why then the difference in conclusion?

The difference in both treatments becomes clear in Eqs. (46,47) [11]. There, the ansatz is transformed into a different restframe, in this case Bob's. This transformation is done making use of the standard Lorentz-transformations (up to higher order corrections). However, looking at the ansatz, one sees that Δx_{AliceQ} is a product of a space-time distance and an energy, here the photon's energy. One knows how these both quantities transform, the transformations have indeed been used in Eqs. (36,37) [11] already. The energy transforms under the DSR transformations, and the space-time distance has then to transform accordingly to allow the invariance of the speed of light in position space.

Consequently, it is the x (or |a| respectively) that is a distance in position space that transforms according to [11] (46,47), whereas with the ansatz [11] Eqs.(44,45) the postulated uncertainty obtains an additional factor from the red/blue-shift of the energy appearing therein. Even to leading order in l_p this is not consistent with the transformation behavior used in [11] (46,47). One obtains instead the transformation behavior for the spread used in [7]. It has been shown there that this is not sufficient to hide the nonlocality in all restframes (see Eqs. (14) - (18) [7]).

It is on the other hand of course possible to enforce the transformation behavior postulated in [11] (46,47). But then the ansatz Eq. (1) is not observer-independent in that it would not preserve its form by a change of reference frame. This ansatz for the uncertainty is, not coincidentally, identical to the difference in arrival times for photons that had been predicted which is cause of the problem. Giving up its observer-independence would mean that the propagation of the photons is either not, as claimed by DSR, observer-independent. Or the ansatz does not agree with the delay in all restframes, which is also not observer-independent.

We thus come to conclude that one can either indeed hide the nonlocality with the ansatz proposed in [11] but this spoils observer-independence. Or one preserves observer-independence, and then one recovers the conclusion previously drawn in [7], that the nonlocality cannot be hidden in all restframes. This is a consequence of the funny transformation behavior of the wave-packet's spread that is forced upon us by observer-independence of the modified dispersion relation. The problem arises from this transformation being different from that of space-time distances defined by other means (the intersection of world-lines), and there is always a restframe in which this inconsistency becomes observable. To quantify the problem properly, at least three worldlines are necessary to obtain sufficient intersections.

It is interesting at this point to have a look at another recent paper [12] by Jacob *et al.* The authors find in this paper the correct transformation of the time-delay from one

restframe into the other. For the case n = 2 considered in [7] they find (see Eq. (30) [12])

$$\Delta t'_{II} = \frac{1+\nu}{1-\nu} \Delta t_I \ . \tag{2}$$

This is indeed the same as Eq.(9) [7].

In [12] it is then moreover correctly concluded that this "poses an immediate challenge for the consistency of this scenario." This inconsistency is interpreted to arise from some "fuzziness" though it is acknowledged later that the arising nonlocality is of "possibly sizeable distan[ce]" and requires "a rather drastic change in the description of spacetime." The problem pointed out in [7] is dismissed (see footnote 7 [12]) with the argument that [7] allegedly assumed that a "novel geometric description of spacetime could at best affect the structure of a spacetime point only locally, in a neighborhood of size $1/E_{QG}$ " (i.e. somewhere close to the Planck scale). In fact, no such assumption has been made in [7]. The bound derived in [7] is based on allowing a change in spacetime structure below the limits that the considered interaction is testing known physics. One can alternatively understand the calculation in [7] the other way round: the necessary "rather drastic" modification of the spacetime picture to reconcile the inconsistencies in the DSR spacetime picture had to be nonlocal on the km scale at least.

Both [12] as well as [11] thus recover the same problem pointed out in [7].

3 And else

While we are at it, let us have look at the rest of [11]. It is comforting, though not particularly surprising, that the spread of the wave-packet found in Eq. (65) [11] is the same as that used in [7], since this a just consequence of the dispersion-relation having a first order contribution in E/m_p together with using a linear superposition. It remains unclear why the author of [11] states that the problem pointed out in [7] is "not surprising, because quantum effects are being treated inconsistently" and then attempts to address the problem by examining the same spread of the wave-function already considered in [7]. It is however good to see and very welcome indeed that this derivation has been done in a straight-forward way starting from the κ -Poincaré algebra.

The way this was achieved is to make the novel observation that the non-commutative space-time coordinates x, t in the κ -Poincaré algebra do not make a good set of observables for quantum mechanics. Instead, it was argued in [11] that one needs to introduce a new time coordinate T, see Eq.(8) [11],

$$T = t + \frac{x^i p_i}{m_p} e^{-E/m_p} , \qquad (3)$$

that commutes with x. Then, x and T form a complete set of commuting observables that can be used to describe the propagation in position space. One then needs to find the transformations from the E, p-basis in momentum space to the x, T-basis in position space.

It is worthwhile to note that if one introduces the T already in Eqs.(17), (18), (19) [11], i.e. before making the transformation from E,k to E,p, then the commutator algebra is simply the standard one, with [T,E] = -i, [x,k] = i and all other brackets vanishing. One should not be fooled to believe that this means the physics is the usual too. The physics is contained in the evolution equation, given by the Hamiltonian (constraint), or the Casimir operator respectively, which in these E,k coordinates, for massless particles, takes the form

$$k^2 = m_{\rm p}^2 \left(1 - e^{-E/m_{\rm p}} \right)^2 \ . \tag{4}$$

The evolution is thus not identical to the usual one. There are two points we should take away from here. One is that the transformation Eq.(3) was constructed with the intent to change the commutation relations and thus does not constitute a canonical transformation². The second is that either way the physics is contained in the evolution equation, not in the commutator algebra.

There is another way than the one taken in [11] to arrive at an expression for the speed of light with this ansatz which is as follows. From the commutator relation Eq. (24) [11] one obtains for the momentum operator in position space

$$\hat{p} = -\mathrm{i}\partial_x e^{\mathrm{i}\partial_T/m_p} \ . \tag{5}$$

To see this, first note that in position space $(\hat{T} = T, \hat{x} = x)$ since [T, E] = -i one has $\hat{E} = i\partial_T$ as usual. We can then compute $x\hat{p}(\cdot) - \hat{p}x(\cdot)$ with Eq. (5) which yields

$$i\partial_{x}e^{i\partial_{T}/m_{p}}x(\cdot) - xi\partial_{x}e^{i\partial_{T}/m_{p}}(\cdot) = ie^{i\partial_{T}/m_{p}}(\cdot)\partial_{x}x = ie^{i\partial_{T}/m_{p}}(\cdot) = ie^{\hat{E}/m_{p}}(\cdot) .$$
(6)

We have used here that the x and T coordinates commute, since they were constructed this way. Eq.(5) thus fulfils the right commutator relation. An easier way to arrive at \hat{p} is to use that k and x fulfill the standard commutation relations, thus $k = -i\partial_x$, and then one uses the definition $p = k \exp(E/m_p)$ to obtain \hat{p} in Eq. (5).

With this momentum operator, the wave-equation takes the form

$$m_{\rm p}^2 \left(1 - e^{-i\partial_T/m_{\rm p}}\right)^2 + \partial_x^2 = 0$$
 (7)

The solutions to this equations are proportional to

$$\exp(i(xk - ET)) \quad , \tag{8}$$

with

$$k = \pm m_{\rm p} \left(1 - e^{-E/m_{\rm p}} \right) \quad , \tag{9}$$

 $^{^2}T$ is in fact not uniquely defined by the requirement that it brings the algebra in a particular form, but only up to a canonical transformation. That freedom in the definition however does not affect the following conclusions about the speed of massless particles.

and the phase velocity is E/k with the above constraint thus

$$c_p(E) = \frac{E}{m_p} \frac{1}{1 - e^{-E/m_p}} \quad ,$$
 (10)

while the group velocity is

$$c_g(E) = \frac{1}{dk/dE} = e^{E/m_p} ,$$
 (11)

which is in agreement with the later derivation of the group-velocity in Eqs. (71) [11] ff³.

Recalling what we noted above, we should not be surprised that the result obtained in [11] differs from the result in [13] since the change of the time-coordinate is not a canonical transformation, and thus the dynamics imposed is physically different in both cases.

Finally, let us look at one of the closing remarks in [11], since it addresses the "worst thing that could happen," that is "if the paradoxes of the classical theory [...] are not resolved in all cases." Since we explained above the worst thing is happening, a comment seems in order. In a nutshell, the argument offered is that the nonlocal interactions required by DSR might not have been observed yet since the location of the particles would have to be very exactly chosen in order for a nonlocal interaction to take place. If we think again about the one point that splits up into three points by a change of reference frame, it might seem that observer-independence would merely require particles on exactly two of these three points to interact with the same probability as the particles in the one point in the original frame. This is indeed an excellent objection, and one that has not been addressed in [7].

However, the argument neglects to take into account that the splitting of the point does not only depend on the phase-space of each individual particle, it does also depend on the distance the particles have travelled. To see this, recall we noted earlier that what causes the splitting of the point to become macroscopically large is what also causes the time-delay of the highly-energetic photon to be observable in the first place: the long distance travelled. Thus, the location of the three points that a change of reference frame creates out of the original one point also depend on this distance. In the more general case it would depend on the distances of all the particles involved in defining the point. The interaction probability had to be the same for all of these points created by all of these distances. Unless there is a not obvious degeneracy in these locations, the interaction points should cover a subset of a 2-dimensional surface. The interaction probability would have to be the same for any two points in that subset, which would vastly increase in-medium (or close-to medium) interactions. In any case, this option might deserve further investigation.

4 Conclusion

The ansatz proposed in [11] for the spread of the wave-packet does not solve the problem discussed in [7]. [12] confirms the same problem, but the authors stop short from

³This calculation has been corrected in the updated version of [11].

quantifying it and instead interpret the inconsistency as the arising necessity for quantum "fuzziness."

Acknowledgements

I thank Lee Smolin and Stefan Scherer for helpful comments. This work was supported by the Queen of Hearts under grant 299792458.

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